$$f_c q = fq + cq = r, \qquad (6)$$

where c is an *arbitrary scalar*, I find that the *new* formula of solution, or of inversion, may be thus written:

$$f_c F_c = n_c\,; \qquad (7)$$

where
$$F_c = F + cG + c^2 H + c^3, \qquad (8)$$

and
$$n_c = n + n'c + n''c^2 + n'''\,c^3 + c^4\,; \qquad (9)$$

G and H being the symbols (or characteristics) of *two new linear operations*, and n', n'', n''' denoting *three new scalar constants*.

8. Expanding then the symbolical product $f_c F_c$, and comparing powers of c, we arrive at *three new symbolical equations*, namely, the following:

$$fG + F = n'\,;\; fH + G = n''\,;\; f + H = n'''\,; \qquad (10)$$

by elimination of the symbols, F, G, H, between which and the equation (5), the *symbolical biquadratic*,

$$0 = n - n'f + n''f^2 - n'''\,f^3 + f^4, \qquad (\text{A})$$

is obtained.

B. B. Stoney, B.A., read the following paper:—

On the Strength of Long Pillars.

Among the numerous difficulties encountered in designing large iron structures, such as railway girders or roofs of large span, none perhaps is of more importance, or requires greater skill to overcome, than the tendency of parts under compression to deflect beneath the pressure, and yield sideways, like a thin walking-cane, when the load is greater than it can support without bending.

To understand the matter clearly, we must recollect that the mode in which a pillar fails varies greatly, according as it is long or short in proportion to the diameter. A very short pillar—a cube, for instance—will bear a weight sufficient to splinter or crush it into powder; while a still shorter pillar—such as a penny, or other thin plate of metal—will bear an enormous weight, far exceeding that which the cube will sustain, the interior of the thin plate being prevented from escaping from beneath the pressure by the surrounding particles. We can thus conceive how stone or other materials in the centre of the globe withstand pressures that would crush them into powder at the surface, merely because there is no room for the particles to escape from the surrounding pressure.

It has been found by experiment that the strength of short pillars of any given material, all having the same diameter, does not vary much, provided the length of the pillar is not less than one, and does not exceed four or five diameters; and the weight which will just crush a short pillar, one square inch in section, and whose length is not less than one or greater than five inches, is called the *crushing strength* of

the material experimented upon. If the length of pillars never exceeded four or five diameters, all we need do to arrive at the strength of any given pillar would be to multiply its transverse area in square inches by the tabulated crushing strength of that particular material. It rarely happens, however, that pillars are so short in proportion to their length; and hence we must seek some other rule for calculating their strength, when they fail, not by actual crushing, but by flexure.

If we could insure the line of thrust always coinciding with the axis of the pillar, then the amount of material required to resist crushing merely would suffice, whatever might be the ratio of length to diameter. But practically it is impossible to command this, and a slight deviation in the direction of the thrust produces a corresponding tendency in the pillar to bend. With tension-rods, on the contrary, the greater the strain, the more closely will the rod assume a straight line, and, in designing their cross section, it is only necessary to allow so much material as will resist the tensile strain. This tendency to bend renders it necessary to construct long pillars, not merely with sufficient material to resist crushing, supposing them to fail from that alone, but also with such additional material or bracing as may effectually preserve them from yielding by flexure. It is evidently, therefore, of considerable importance that we should ascertain the laws determining the flexure of long pillars, which may be done as follows:—

Let the figure represent a pillar, very long in proportion to its breadth, and just on the point of breaking from flexure.

Let W = the deflecting weight;
b = the breadth of pillar;
d = its depth;
l = its length;
h = the central deflection;
R = the radius of curvature;
C = the resultant of all the longitudinal forces of compression on the concave side at the centre of the pillar;
T = the resultant of all the longitudinal forces of tension on the convex side;
δ = the distance between the centres of tension and compression.

The longitudinal forces acting at the centre of the pillar are three, viz. the weight W acting in the chord line of the curve, the resultant C acting at the centre of compression in the concave half, and the resultant T acting at the centre of tension in the convex half. Taking moments round either centre of strain, we have approximately

$$W = \frac{T\delta}{h} = \frac{C\delta}{h}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{I.}$$

h being assumed equal to the distance between the chord-line and either

centre of strain, which is a close approximation when the pillar is very long in proportion to its width.*

The values of T or C in different pillars are proportional to the number of fibres subject to strain, that is to bd, and δ is obviously proportional to d; so that we have the numerator on the right side of the equation proportional to bd^2. Again, assuming that the deflection curve is a parabola, from which it can differ but slightly,† we have

$$h = \frac{l^2}{8R};$$

but so long as the strain per sectional unit in the extreme fibres, to which their change of length is proportional, is constant, R will vary in the same ratio as d; and we have, therefore, h proportional to

$$\frac{l^2}{d}.$$

Whence, by substitution,

$$W = K\frac{bd^3}{l^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II.}$$

in which K is a constant depending on the elasticity of the material, which may be determined by experiment.

If the pillar be round, and if d represent the diameter,

$$W = K\frac{d^4}{l^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{III.}$$

which proves that the strength of long round pillars varies as the 4th power of their diameter, divided by the square of the length; and the longer the pillar is in proportion to its diameter, the nearer will this formula represent the truth.

As all the longitudinal forces at the middle of the pillar balance, we have the following equation:—

$$C = T + W,$$

which enables us to predict how a long pillar will fail, whether by the convex side tearing asunder, or by the concave side crushing. A wrought iron pillar, for instance, may be expected to fail on the concave side, as its power to resist crushing is less than that to resist extension. A long pillar of cast iron, on the contrary, will probably fail by the convex side tearing asunder, as the compressive strength of cast iron greatly exceeds its tenacity. Further, the effective strength of wrought iron to resist crushing is about 12 tons per square inch, while the tensile strength of cast iron is nearly 7 tons per square inch; and hence we

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* Mr. Hodgkinson's experiments show that this investigation is not applicable to cast iron pillars whose length is less than about 30 times their width: even with such short pillars it requires certain modifications, which he has deduced from experiment.

† The curve will probably be intermediate between a parabola and a circle, approaching the latter if the pillar taper towards the ends.

may conclude that the strength of long similar pillars of wrought and cast iron will be nearly as 12 to 7.

It is also worthy of note that, if the same pillar be bent in different degrees, T will vary as h, while δ remains constant; whence it follows from equation (I.) that W, the weight which keeps the pillar bent, is nearly the same whether the flexure be greater or less. This statement would be accurately true, were it not that equation (I.), on which it is founded, is only approximate. It will, however, agree very closely with experiment so long as h is considerable, that is, whenever the flexure is not slight. From this it follows, that any weight which will produce considerable flexure will be very near the breaking weight, as a trifling addition to it will bend the pillar very much more, and strain the fibres beyond what they can bear.

The Secretary of Council, for Hodder M. Westropp, Esq., read a paper—

On the Fanaux de Cimitieres and the Round Towers.

In reading De Caumont's "Rudiments d'Archeologie," I have been struck with a remarkable analogy between the Irish Round Towers and what are named in De Caumont's work "Fanaux de Cimitieres," and also "Lanterns of the Dead." The following is his description of them:—

"Fanaux de Cimitieres are hollow towers, round or square, having at their summit several openings, in which were placed, in the middle ages (twelfth and thirteenth centuries), lighted lamps, in the centre of large cemeteries. The purpose of the lamp was to light, during the night, funeral processions which came from afar, and which could not always reach the burial-ground before the close of day. The beacon, lighted, if not always, at least on certain occasions, at the summit of the towers, was a sort of homage offered to the memory of the dead—a signal recalling to the passers-by the presence of the departed, and calling upon them for their prayers. Mr. Villegille has found in Pierre de Cluni, who died in 1156, a passage which confirms my opinion. These are the words in which he expresses himself with regard to the small tower of the beacon of the monastery of Cherlieu:—'Obtinet medium cemiterii locum structura quædam lapidea, habens in summitate sui quantitatem unius lampadis capacem, quæ ob reverentiam fidelium ibi quiescentium, totis noctibus fulgore suo locum illum sacratum illustrat.'

"Mr. Lecointre Dupont remarks, that these towers or beacons are found particularly in cemeteries which were by the side of high-roads, or which were in greatly frequented places. 'The motive for erecting these beacons was,' he says, 'to save the living from the fear of ghosts and spirits of darkness, with which the imagination of our ancestors peopled the cemeteries during the night-time; to protect them from that *timore nocturno*, from that *negotio perambulante in tenebris* of whom the Psalmist speaks; lastly, to incite the living to pray for the dead.'